Formulas for TVI curves

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Abstract

This paper presents a reformulation of a set of polynomials that can be used to generate TVI curves. The reformulation initially looks perhaps a bit unfamiliar, since it is not in the standard "canonical form". However, this reformulation has some advantages over the standard form.

One of the benefits of the reformulation is to be more readily interpreted. A TVI curve can be numerically described as having coefficients for the 50% TVI, for the "lean" of the curve, and for the "bulge" of the curve. These three coefficients are expressed directly in the formula so that they can be readily determined.

A second benefit of the reformulated polynomials is that adjustments of the TVI curves are easier to compute. In particular, changing from, for example, a 21% TVI to a 22% TVI requires changing a coefficient from 0,21 to 0,22. If the TVI curve needs to lean more to the left, the lean coefficient is made a bit more negative. If the TVI curve needs to be fatter, the bulge coefficient is increased,

A third benefit of the reformulated polynomials is that they lend themselves to a mathematical technique for least squares curve fitting. Standard polynomial regression techniques, such as those available in Excel, will not generally provide a polynomial that goes exactly through zero at a tone value of 100%. The reformulated polynomials are guaranteed to be exactly zero at 0% TV and 100% TV. While Excel does not provide a simple function for doing regression with the reformulated polynomials, the necessary calculations can be implemented in Excel.

Relevant parts of the standards

The most recent draft of 12647-2 has provided TVI formulas, in addition to the plots and tabulation of the target TVI at a number of specific tone values. See ISO 12647-2, section 4.3.4.1, Note 5, of N 1060.

Providing formulas for the TVI curves is useful in that target TVIs may be determined for tone values not included in the table. I would like to propose that the formulas be presented in a way that is perhaps a little more user friendly and readily calculated. In the first section of this paper, I present the language that appears in the current draft, and my proposed alternative that is mathematically equivalent to the current draft.

In the second section of this paper, I provide a proposal for an annex to describe the benefits of the different approach.

The third section of this paper is more of a theoretical nature. While the accompanying spreadsheet provides a way to perform the regression, this third section describes the theory so that this could be incorporated into a piece of software.

Language for section 4.3.4.1, note 5, 12647-2

Existing language (N 1060 draft)

NOTE 5 For process calibration and control purposes it is sometimes necessary to calculate the tone value or tone value increase on a print for additional tone values. For this an exemplary fourth order polynomial function describing the curves in Figure 2 is given as follows:

$$TVI(x) = 100*(a*x^4 + b*x^3 + c*x^2 + d*x)$$

where

TVI	is the tone value increase as a percentage value;
a, b, c, d	are the coefficients of the polynomial;
x	is the tone value normalized between 0 and 1; x = TV/100;
TV	is the tone value in % ranging from 0 to 100;

The polynomial coefficients are given in Table 7.

Table 7 — Polynomial coefficients for tone value increase curves in Figure 2

Polynomial Coefficient	Tone Value Increase Curve				
	A	B	C	D	E
a	-0,3650	-0,5877	-0,7854	-0,4441	-0,0438
b	0,6730	1,3575	1,9934	1,4386	0,7664
c	-1,0108	-1,7678	-2,4956	-2,3805	-2,1929
d	0,7029	0,9980	1,2876	1,3860	1,4703

Proposed alternative language

Note 5 For process calibration and control purposes it is sometimes necessary to calculate the tone value increase aim values for additional tone values. For this an exemplary equation describing the curves in Figure 2 is given as follows:

$$TVI(x) = a * p_1(x) + b * p_2(x) + c * p_3(x)$$

where

TVI is the tone value increase as a percentage value;

p_1, p_2, and p_3 are fundamental TVI curves, described below;

a, b, c are the coefficients of p_1, p_2, and p_3, representing TVI, lean, and bulge, respectively.

x is the tone value, normalized between 0 and 1; that is, x = TV / 100;

TV is the tone value in % ranging from 0 to 100; and

$$p_1(x) = -4 * x * (x - 1)$$

$$p_2(x) = 21 * x * (x - 1) * \left(x - \frac{1}{2}\right)$$

$$p_3(x) = -64 * x * (x - 1) * \left(x - \frac{1}{2}\right)^2$$

The coefficients are given in Table 7.

Table 7 – Coefficients for tone value increase curves in Figure 2

Coefficient	Tone value increase curve				
	A	B	C	D	E
a (TVI)	16,0	19,0	22,0	25,0	28,0
b (lean)	-0,3	0,9	2,0	2,6	3,2
c (bulge)	0,6	0,9	1,2	0,7	0,1

Language for 12647-3

Similar language should appear in the note in section 4.3.5.1. Table 6 should be changed so that TVI (the coefficient a) should be 26, lean (the coefficient b) should be 2.3, and bulge (the coefficient c) should be 0.

Proposed annex

Annex XX

(informative)

Fundamental TVI curves

The so-called "fundamental TVI curves" described in the body of this document are perhaps a bit unfamiliar, so they deserve a bit of explanation. The curves could be described in the standard canonical form for a fourth order polynomial. Indeed, these are just reformulations of fourth order polynomials. For example, the equation for TVI curve E in Table 7 is

$$TVI(x) = 28 * \left(-4 * x * (x-1)\right) +$$

$$3{,}2 * \left(21 * x * (x-1) * \left(x - \frac{1}{2}\right)\right) +$$

$$0{,}1 * \left(-64 * x * (x-1) * \left(x - \frac{1}{2}\right)^2\right)$$

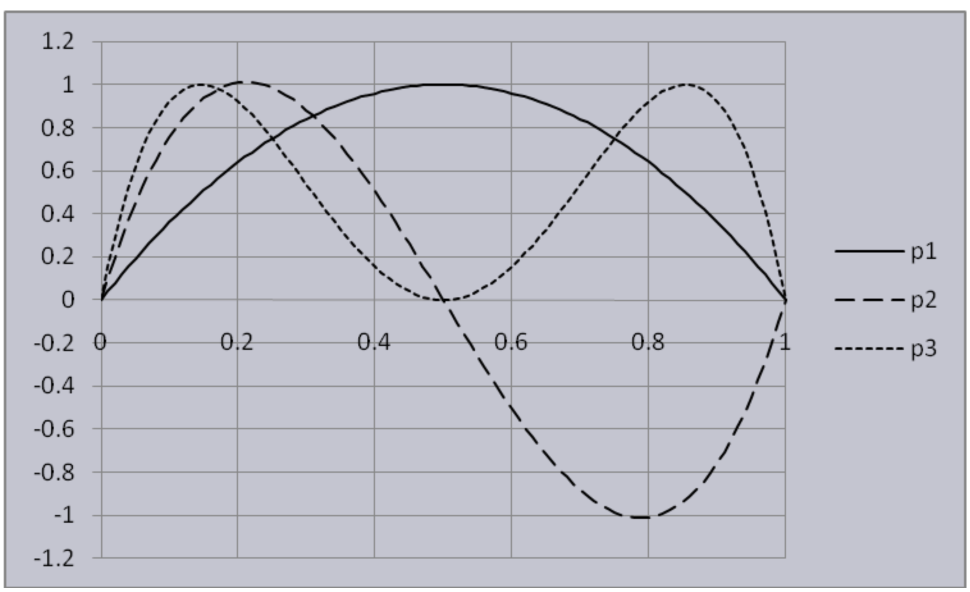
This could have been written more concisely as

$$TVI(x) = -3{,}2x^4 + 74{,}02x^3 - 217{,}43x^2 + 146{,}61x$$

The two equations are mathematically equivalent. Although the first formulation is more typing and at first glance appears more complicated, there are a number of advantages to this longer form.

Figure XX shows the three fundamental TVI curves, $p_1(x)$, $p_2(x)$, and $p_3(x)$. The first thing to note is that all three of the polynomials are zero at both $x = 0$, and $x = 1$. This assures that the TVI curve will be zero at these points regardless of the choice of coefficients.

Figure XX – The three fundamental TVI curves



As can be seen, the first curve, $p_1(x)$, has the general overall parabolic shape of a TVI curve. Since the other two curves are designed to be zero at $x = 0{,}5$, the coefficient of this curve is identical to the TVI at 50%. In the example above, the TVI at 50% is quite readily seen to be 28%. If a change in TVI at 50% is required, this may be accomplished by changing the first coefficient.

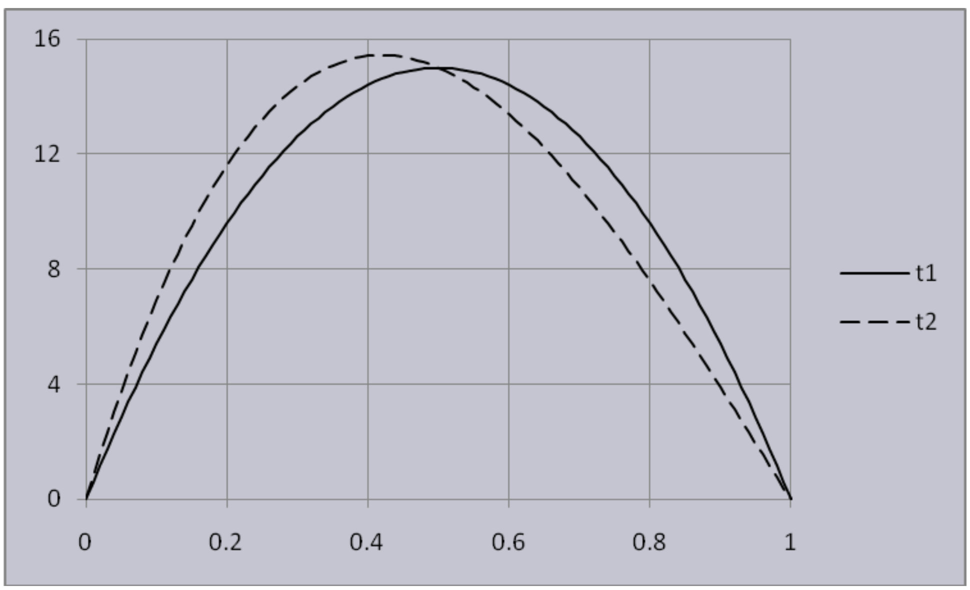
The second fundamental TVI curve, $p_2(x)$, defines the lean of the curve. If the value is this coefficient is positive, then the TVI curve will lean to the left. A left-leaning TVI curve will reach a maximum TVI will below a tone value of 50%. A negative value will cause the maximum to occur above 50%. The larger the second coefficient (in magnitude) the farther the maximum will move from 50%. This second coefficient is analogous (but not equivalent) to the statistical term "skewness".

Figure XX is an example of the effect of adding lean to the TVI. The curve t1 is the TVI curve generated with a TVI of 15 with no lean parameter. The curve t2 is the TVI curve with a lean parameter of 2.0. That is to say,

$$t_1(x) = 15 * p_1(x)$$

$$t_2(x) = 15 * p_1(x) + 2 * p_2(x)$$

Figure XX – The affect of the lean parameter



Note that adding in lean will change the maximum TVI. In this example, the maximum TVI is 15.42 with the lean parameter added in, as opposed to 15 without this parameter. The amount of TVI at 50%, however, remains constant.

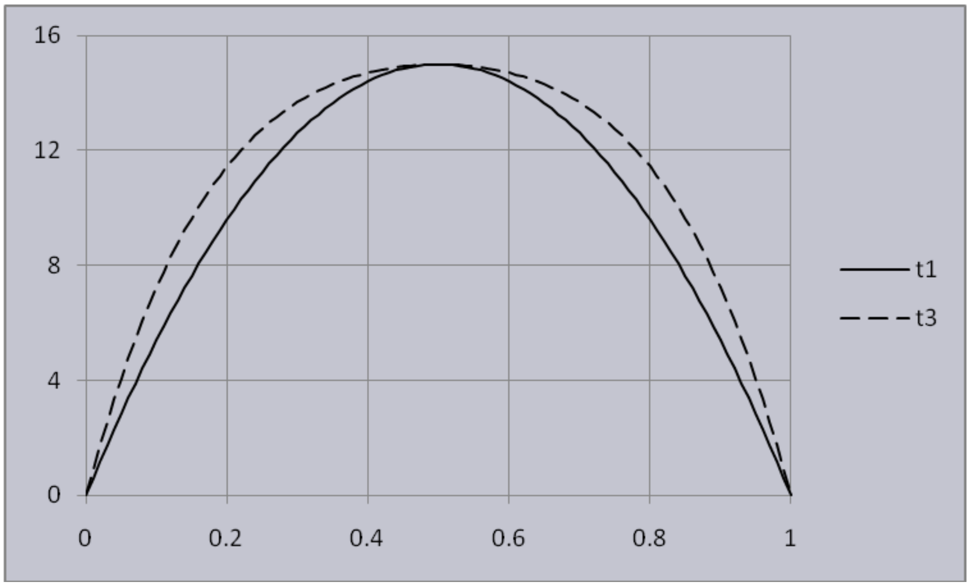
The third fundamental TVI curve, $p_3(x)$, defines the bulge of the TVI curve. If the bulge coefficient is positive, the resulting TVI curve will bulge out more than a parabola. A negative bulge coefficient will not bulge out quite as much as a parabola. The bulge coefficient is analogous (but not equivalent) to the statistical term "kurtosis".

Figure XX is an example of the affect of adding bulge to a TVI curve. The curve t1 is, as in the previous example, a 15% TVI curve with no lean or bulge. The curve t2 shows the affect of adding a bulge of 2.

$$t_1(x) = 15 * p_1(x)$$

$$t_3(x) = 15 * p_1(x) + 2 * p_3(x)$$

Figure XX – The affect of the bulge parameter



The fundamental TVI curves are designed to have a maximum of very close to 1. This is reflected in the choice of -4, 21, and 64 as coefficients buried in the formulas. (The curve $p_2(x)$ has a maximum of roughly 1,01. The benefit of having this be exactly 1,00 was weighed against the cleanliness of having the number 21 in the formula instead of 20.7846.)

Since the maximum of the curves is 1,00, it is easy to look at the coefficients of $p_1(x)$, $p_2(x)$, and $p_3(x)$ to determine the magnitude of the affect of each. For example, in the equation for TVI curve E in Table 7, it can be seen that the lean parameter, which is 3,2, will change the TVI by no more than 3,2%. The bulge parameter, 0,1, is insignificant, since it can't change the TVI by more than 0,1%.

Solving for coefficients via regression

-- This section is additional background material, and may not belong in the standard. --

The idea of linear regression is perhaps narrowly understood by the scientific community. As it is commonly used, it refers to the act of finding a set of coefficients for a standard polynomial that provide the best match to a set of data.

This is one form of linear regression. It may appear that this is a misnomer, since x^2 and x^3 are not linear equations. One may have wondered whether the phrase was misappropriated, and should only be applied to the act of finding the coefficients *m* and *b* in the linear equation $y = mx + b$.

The phrase linear regression is properly used to refer to polynomial regression, but is perhaps misunderstood. More broadly, linear regression refers to the act of finding the *linear combination* of a set of basis functions so as to give the best fit to a set of data. Linear combination means that each of the functions is to be multiplied by a coefficient and the products are then to be added together to produce the final curve. Linear regression determines the "best" set of coefficients.

The simplest example of linear regression – fitting a line to data – uses the functions "1", and x as the basis functions. For polynomial regression, the basis functions are $1, x, x^2, x^3, \ldots$ This is linear regression because one is trying to find a linear combination of these functions.

Although it is not generally thought of this way, the Fourier transform is another example of linear regression. In this case, the basis functions are not polynomials, but are sine and cosine functions.

Another example of linear regression is multiple linear regression, in which there are a number of independent variables and one wishes to find the linear combination of these parameters which gives the best fit to a set of dependent data. For example, one may wish to use multiple linear regression to describe the L* value as a function of the cyan, magenta, yellow, and black tone values.

For this paper, the basis functions are $p_1(x)$, $p_2(x)$, and $p_3(x)$.

$$p_1(x) = -4 * x * (x - 1)$$

$$p_2(x) = 21 * x * (x - 1) * \left(x - \frac{1}{2}\right)$$

$$p_3(x) = -64 * x * (x - 1) * \left(x - \frac{1}{2}\right)^2$$

These basis functions were chosen so that they all equal are to zero at a tone value of 0 and at tone value of 100%. Since this is the case, all linear combinations of them will also go through zero at these points.

The second and third basis functions ($p_2(x)$, and $p_3(x)$) were chosen to be zero at a tone value of 50%. In this way, the coefficients of these two basis functions will not effect that 50% tone value increase.

The second basis function was chosen to be anti-symmetric about a tone value of 50%. In this way, it could serve a purpose similar to the skew parameter in statistics. The third basis function was chosen to mimic the kurtosis parameter.

When taken together, any linear combination of the three basis functions is a fourth order polynomial which is zero. The converse is also true. All fourth order polynomials which are zero at $x = 0$ and at $x = 1$ can be expressed as a linear combination of these basis functions.

What all of these examples of linear regression have in common is that they can be solved with the same general mathematical technique. Suppose one has decided that the basis functions that were previously described ($p_1(x)$, $p_2(x)$, and $p_3(x)$) are the appropriate ones to use. And suppose that one has a set of dot gain data that looks like this:

TV	TVI
0,20	9
0,40	13
0,60	12
0,80	8

I evaluate each of the basis functions at each of the tone values, and assemble them into the following matrix.

$$A = \begin{bmatrix} p_1(0{,}20) & p_2(0{,}20) & p_3(0{,}20) \\ p_1(0{,}40) & p_2(0{,}40) & p_3(0{,}40) \\ p_1(0{,}60) & p_2(0{,}60) & p_3(0{,}60) \\ p_1(0{,}80) & p_2(0{,}80) & p_3(0{,}80) \end{bmatrix}$$

Next, I put together a column vector of the TVI values.

$$b = \begin{bmatrix} 9 \\ 13 \\ 12 \\ 8 \end{bmatrix}$$

Finally, we create a column vector of the weights to be applied to each of the basis functions. The values in this vector are unknown at this point. They are what I am trying to find.

$$x = \begin{bmatrix} TVI \\ lean \\ bulge \end{bmatrix}$$

We are trying to find a least squares solution to the following equation. That is, we are trying to find values for the three variables in the b vector that come closest to making this an equality.

$$Ax = b$$

The least squares solution is this:

$x = (A^T A)^{-1} A^T b$

The symbols in the equation are the standard symbols for linear algebra. “T” means transpose, which is inverting the rows and columns of the matrix. “-1” means taking the inverse of the matrix.

The delightful thing about this least squares equation is that it is perfectly general. I can any number of basis functions, and those basis functions can be whatever functions I want. For example, if the basis functions are “1”, and x, I can use this to derive the formula for linear regression. If I include x^2, and x^3, I can use this to derive the formula for cubic polynomial regression. As another example, if those basis functions are sine and cosine functions, I can perform a Fourier transform for frequency analysis.

This formula can also be extended to multiple linear regression. Let’s say, for example, we wanted to investigate how TVI changed with changes in density. In this case, one of our basis functions might be solid ink density, another might be tone value, and a third might be the product of the two.

The only limitation to this approach is that we can only use this when the values of the basis functions can be computed directly from the independent variables. That is to say, the left hand side of the equation $Ax = b$ can’t depend on any of the unknown parameters in the vector b. An example of where we can’t use the formula is where one of our basis functions is the exponential decay function, e^{-kx}, when we are trying to find the decay rate *k*.